

SYNTHESIS OF COMMENSURATE COMB-LINE BAND-PASS FILTERS WITH
HALF-LENGTH CAPACITOR LINES, AND COMPARISON TO EQUAL-LENGTH
AND LUMPED-CAPACITOR CASES

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Abstract

Comb-line band-pass filters have previously been synthesized as circuits having equal-length short-circuited and open-circuited TEM lines. Practical designs, however, usually use almost-lumped capacitors. This paper presents an exact synthesis method for commensurate circuits in which the capacitor lines are exactly half the length of the inductor lines. Computed examples show that the pass-band response and stop-band skirts agree much closer to that of lumped and almost-lumped capacitors than that of the equal-line-length case. Also, a remarkable improvement in spurious response degradation will occur. Implementation of the half-length design is explained and is rather simple to carry out.

Summary

Comb-line band-pass filters are widely used in the microwave range. Design may be carried out approximately¹ when the bandwidth is narrow (up to about 20%), while for any bandwidth its idealized TEM-line equivalent circuit has been synthesized exactly, subject to an equal-line-length assumption.² This restriction requires the capacitors to be open-circuited lines equal in length to the short-circuited inductor lines. In practice, the capacitors are usually shorter than the inductors and often are almost-lumped elements. Reference 2 shows how a synthesized equal-length design can be altered to apply to lumped-capacitors, achieving correct response at the pass-band edges, but deviating from ideal within the pass-band.

The principal contribution of this paper is to show how exact synthesis may be achieved with capacitor lines exactly half the length of the inductor lines. This ratio is convenient in practice, and furthermore, when applied to shorter capacitors, the deviation from the ideal pass-band is much smaller than with designs based on equal-line-length capacitors. Also, the stop-band skirts behave more nearly as expected and spurious responses near 180° are eliminated.

Figure 1(a) shows the comb-line configuration with equal-length capacitors and Figure 1(b) shows half-length capacitors. The indicated terminal connection to the end resonators is appropriate for medium and wide bandwidths. Other termination techniques suitable for narrower bandwidths are available^{1,2,3} and need not be discussed here.

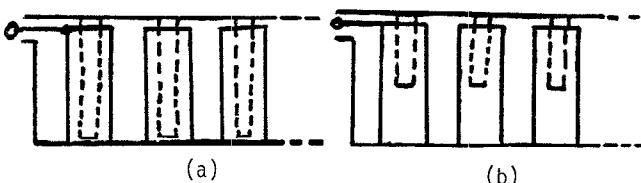


Figure 1. Comb-line band-pass filters with full and half-length capacitor lines

Commensurate TEM-line equivalent circuits for the cases in Figure 1 appear in Figure 2. One should remember that the term commensurate is not restricted to equal-line-lengths, but allows unequal lengths as long as they are integral multiples of a common sublength. Richard's synthesis theorems apply to commensurate circuits in this broad sense.

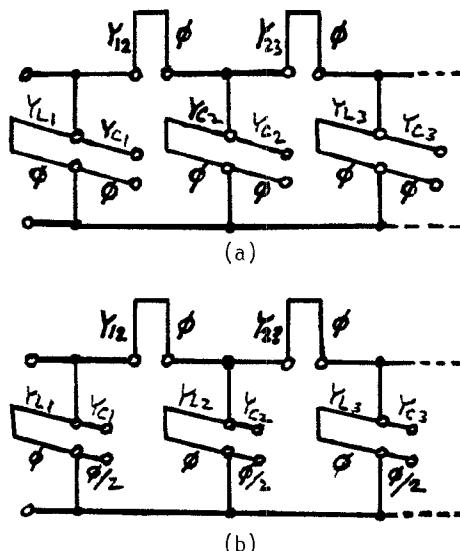


Figure 2. Transmission-line diagrams of cases in Figure 1.

The equivalent circuit shown in Figure 3(a) is used for the equal-line-length schematic shown in Figure 2(a). Note the relations between L, C, the characteristic admittances and the replacement of $\omega = 2\pi f$ by $\Omega = \tan \phi$. For the half-length capacitors, equivalence in Figure 3(b) requires a different relation set and replacement of ω by $\Omega = \tan(\phi/2)$. These relations yield exact equivalence.

The rationale behind Figure 3(b) is justified as follows: Consider the identity

$$\cot \phi = \frac{1}{2} \cot \frac{\phi}{2} - \frac{1}{2} \tan \frac{\phi}{2}$$

This causes the susceptances of the shorted lines to be

$$-Y_{Li} \cot \phi = \frac{1}{2} Y_{Li} \cot \frac{\phi}{2} + \frac{1}{2} Y_{Li} \tan \frac{\phi}{2}$$

and

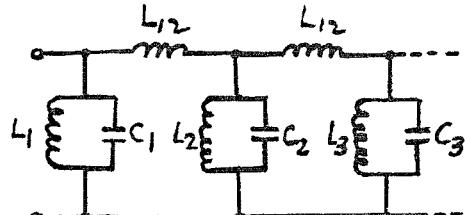
$$-Y_{i,i+1} \cot \phi = \frac{1}{2} Y_{i,i+1} \cot \frac{\phi}{2} + \frac{1}{2} Y_{i,i+1} \tan \frac{\phi}{2}$$

while the susceptances of the capacitive lines are

$$Y_{Ci} \tan \frac{\phi}{2}$$

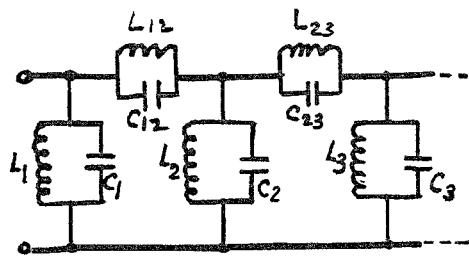
When coefficients of $\Omega = \tan \frac{\phi}{2}$ and $1/\Omega = \cot \frac{\phi}{2}$ are grouped, the relations in Figure 3(b) follow.

Clearly, the L and C circuit shown in Figure 3(b) has a quasi-elliptic response. It cannot be converted to true elliptic-function response, because the series arms all provide rejection at the frequency where $\phi = 90^\circ$. Modification of the structure can shift these rejection frequencies, but this is not included in the scope of this paper.



$$\begin{aligned}\Omega &= \tan \phi \\ C_i &= Y_{Ci} \\ L_i &= Z_{Li} = 1/Y_{Li} \\ L_{i,i+1} &= Z_{i,i+1}\end{aligned}$$

(a)



$$\begin{aligned}\Omega &= \tan \frac{\phi}{2} \\ C_i &= Y_{Ci} + \frac{1}{2} Y_{Li} \\ L_i &= 2 Z_{Li} \\ C_{i,i+1} &= \frac{1}{2} Y_{i,i+1} \\ L_{i,i+1} &= 2 Z_{i,i+1}\end{aligned}$$

(b)

Figure 3. L,C circuits applying to Figures 1 and 2

In addition to the two cases of full and half-length capacitors, a third case has lumped-capacitors, as shown schematically in Figure 4. Comparisons of the pass-band and stop-band response of these three cases are made in Figure 5. Note that the full-length capacitor case has a periodic response with period 180° and symmetry around 90° . For the half-length capacitors, the period is 360° with symmetry around 180° . The response of the lumped-capacitor case is not periodic; its higher pass-bands become narrower in successive order.

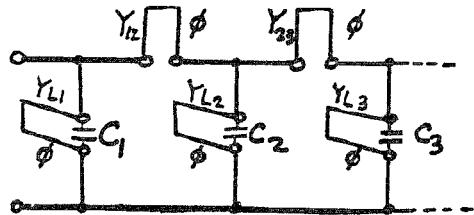
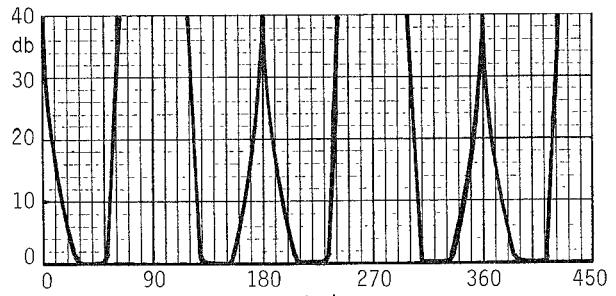
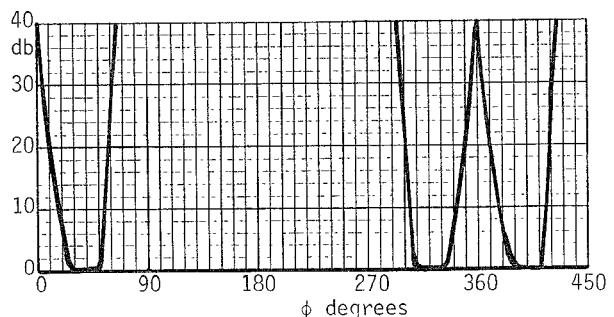


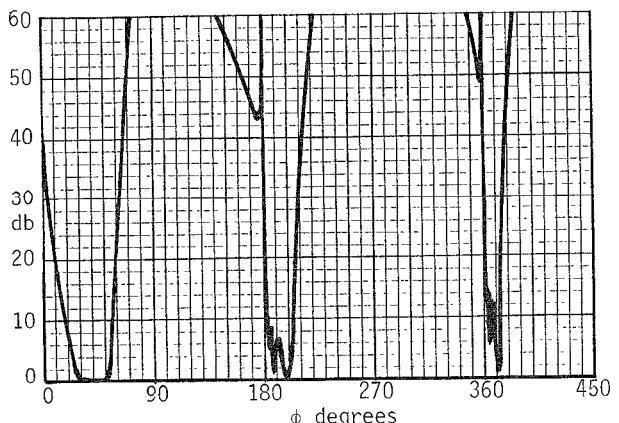
Figure 4. Transmission-line diagram for lumped-capacitor case



(a) Commensurate, equal length



(b) Commensurate, half-length capacitors



(c) Commensurate, lumped capacitors

Figure 5. Comparison of insertion loss response for three $\eta=4$ cases

Four-resonator examples with equal-ripple VSWR = 1.1000 and pass-band edges at ϕ equal to 30° and 50° were used. The lumped-capacitor design cannot be synthesized by rigorous mathematics and therefore was synthesized by a rapidly convergent optimization method applicable in all cases, including mixed-parameter filter circuits.³ The half and full-length-capacitor designs may be synthesized rigorously or by optimization.

All three examples have equal-ripple pass-bands. An interesting comparison occurs when the shunt resonators of the half and full-length capacitor cases are replaced by resonators having lumped-capacitors and altered Y_{Li} such that their shunt susceptances equal their original values at ϕ_1 and ϕ_2 . This ensures correct VSWR at the pass-band edges, but not within the pass-band. Figure 6 shows the pass-band VSWR of altered cases compared to the case synthesized for lumped-capacitors. The response for the altered half-length capacitor case agrees with the lumped-capacitor synthesis case far more closely than does that of full-length capacitors.

References

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- (2) R. J. Wenzel, "Synthesis of combline and capacitively loaded interdigital bandpass filters of arbitrary bandwidth," IEEE Trans. vol. MTT-19, Aug. 1971, pp. 678-686.
- (3) S. B. Cohn, "Generalized design of band-pass and other filters by computer optimization," 1974 IEEE S-MTT International Microwave Symposium, Atlanta, GA, June 12-14, 1974; Digest pp. 272-274.

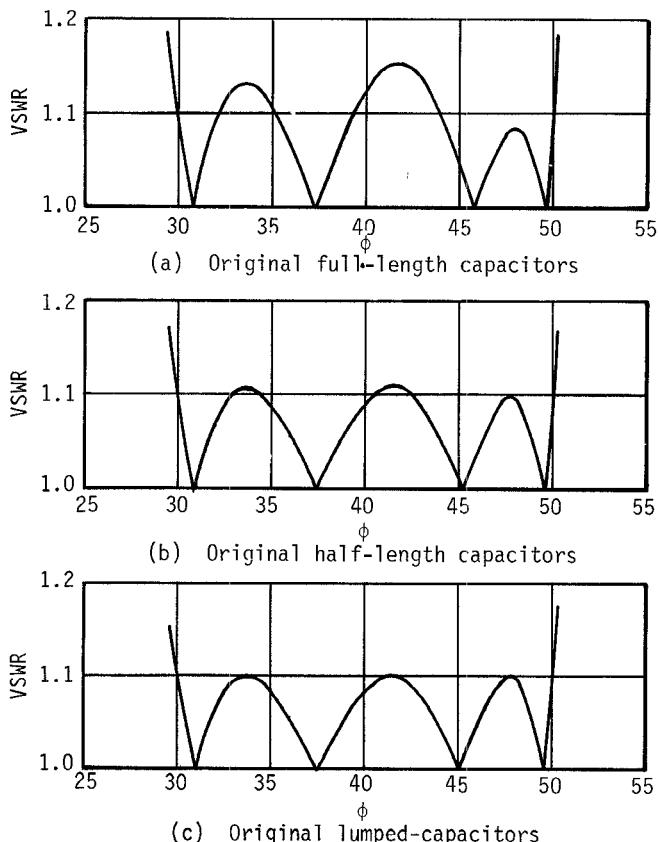


Figure 6. Three examples of Pass-band VSWR with lumped-capacitors

In conclusion, a combline filter, synthesized with half-length capacitors, provides a pass-band more nearly equal to that of practical lumped and quasi-lumped-capacitor filters than does the full-length-capacitor synthesis. It also provides an improved stop-band. Furthermore, the labor involved in synthesizing the half-length design is scarcely more than that for the full-length design.